A Volume Flexible Inventory Model
with Trapezoidal Demand Under Inflation

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Abstract

This article explores an economic production quantity model (EPQ) model for deteriorating items with time-dependent demand following trapezoidal pattern taking the volume flexibility into account. We have also considered the inflation and time value of money. The solution of the model aims at determining the optimal production run-time in order to maximize the profit. The model is also illustrated by means of numerical experiment. Further, the effects of different parameters are analysed by performing sensitivity analyses on the optimal policy.

Keywords: Inventory, Trapezoidal demand, Volume flexibility, Inflation.

1. Introduction

Generally, the researchers assumed the time varying demand as an increasing or decreasing function of time, however in reality; this hypothesis is not appropriate for all products. One can observe the two-fold ramp type pattern in demand for items like fashion apparel, particular kind of eatables and festival accessories having restricted phase for sales and becomes out of use at end of phase. Such structure is known as ''trapezoidal ramp-type''. In opening phase, the demand rises up to a certain time point and becomes constant later but starts falling towards the end phase. Hill (1995) was the first to consider ramp type demand through inventory modelling. After that, various researchers have drawn considerable attention to exploration of ramp-type demand. Mandal and Pal (1998) proposed the inventory model with ramp-type demand for exponentially deteriorating items with shortages. Wu and Ouyang (2000) developed EOQ models for different replenishment policies; one of which included shortage at beginning while the second considered shortage at later phase. Continuing in same manner, Wu (2001) investigated a model for deteriorating items with ramp-type demand and partial backlogging. Giri et al. (2003) studied an extension for ramp-type demand inventory model with Weibull distribution deterioration rate. Manna and Chaudhuri (2006) developed a production model with ramp-type two time periods regarded as demand form taking demand dependent production into account. Deng et al. (2007) got a note published on the uncertain results found by Mandal and Pal (1998) and Wu and Ouyang (2000) and proved a more reliable solution. Panda et al. (2008, 2009) worked on Giri et al.'s (2003) model to change one-fold demand model to two-fold demand. Hill's (1995) model was extended to trapezoidal-type demand rate by Cheng and Wang (2009). Panda et al. (2009) developed the inventory model with quadratic ramp-type demand function which determined the optimal production stopping time. Skouri et al. (2009) extended the model of Deng et al. (2007) to wide-ranging ramp-type demand rate, Weibull distribution deterioration rate, and general partial backlogging rate. Further, this model was extended by Hung (2011). He incorporated arbitrary component in ramp-type demand pattern. Shah and Shah (2012) investigated a model for items having trapezoidal demand in an integrated system.

Inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision making. At present, it is impossible to ignore the effects of inflation and it is necessary to consider the effects of inflation on the inventory system. The application of this concept was initiated by Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. After that, several researchers like Brahmbhatt (1982), Chandra and Bahner (1985), Datta and Pal (1991), Hariga (1995, 1996) and Ray and Chaudhuri (1997) explored the effect of inflation and time value of money on inventory models. Liao et al. (2000) developed an inventory model for initial-stock dependent consumption rate with permissible delay in payment under the inflationary environment. Moon et al. (2005) developed the model for ameliorating/deteriorating items with time varying demand pattern over a finite planning horizon and explored the effects of inflation and time value of money. Jolai et al. (2006) studied the system of perishable items for stock dependent demand and shortages with inflation. Dey et al. (2008) developed a finite time horizon inventory problem for deteriorating items for a two-warehouse system under inflation and time value of money. Roy et al. (2009) worked on inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. Yang et al. (2010) discussed a model under similar environment relaxing the warehouse situation. Das et al. (2011) developed an economic production lot size model for an item with imperfect quality by considering random machine failure. Recently, Guria et al. (2013) worked on a deterministic and random planning horizon based model under inflation considering both the cases of shortage as well as no shortage.

Almost all the studies cited above, considered the constant rate of production. However, constant production rate is not always practical. For instance, as production model is based on time dependent demand, the postulation of constant production rate is inappropriate. Such situation leads to the relevance of variable production rate. Schweitzer and Seidmann (1991) were the first to consider changeable machine production rate. Khouja (1995) proposed an EPQ model with unit production cost based on used raw materials, occupied labor and tool wear and tear cost. Bhandari and Sharma (1999) considered the marketing cost in addition to formulating a comprehensive cost function. The related studies done by Sana et al. (2007) and Sana (2010) may be noted. Dem and Singh (2012) developed a volume flexible model for damageable items with time dependent demand. Dem and Singh (2013) investigated an EPQ model with volume flexibility under imperfect production process. Goyal et al. (2013) developed a volume flexible model with ramp type demand for ameliorating items.

In the present article, an EPQ model for deteriorating items with trapezoidal type demand rate and volume flexibility has been explored. We also assume that the inventory system is affected by inflation and time value of money. Trapezoidal demand rate is usually noticed in the case of seasonal goods. The demand rate for such items intensifies with the time up to certain level and then stabilizes but in final phase, the demand rate declines to a constant or zero. This kind of demand rate is very logical and hence, we purpose a

realistic inventory replenishment policy for such inventory model. The remaining paper is planned as follows. In Section 2, we explain the assumptions and notation used throughout the modelling. In Section 3, we formulate the mathematical model and. In Section 4, the necessary conditions to find an optimal solution are described. We provide numerical example for each case to illustrate the model in section 5. Finally, the conclusion is given in section 6.

2. Assumptions and Notations

2.1. Assumptions

1. Demand rate is dependent on time given by

$$D(t) = \begin{cases} a + b_1 t, & 0 \le t \le t_u \\ a + b_1 t_u, & t_u \le t \le t_v \\ a - b_2 t, & t_v \le t \le T \end{cases}$$

where $a, b_1, b_2 > 0$

The above function is known as trapezoidal function.

2. Production rate is k times demand rate, where k >1.
3. The unit production cost is dependent on production.
4. Time horizon is finite.
5. Deterioration rate is a constant.
6. The deterioration occurs when the item is effectively in stock.
7. The inflation and time value of money has been taken into consideration.

2.2. Notations

$D(t)$ Demand rate

$P(D(t))$ Production rate, $P(D(t))=kD(t)$

θ Deterioration rate

C_1 Set up cost

C_2 Holding cost per unit per unit time

S_o Selling price per unit

$C(P)$ Production cost per unit given by $C(P) = R + \dfrac{G}{kD(t)}$

R Material cost per unit

G Factor associated with costs like labor and energy costs

$I(t)$ Inventory level at any time t

t_u Time when demand becomes constant

t_v Time till the demand remains constant

t_r Time when the production stops

T Total cycle length

3. Model Formulation

We have considered the following different cases based on the occurrence of time points of demand in different phases.

3.1. Case (I) When $t_v < t_r$

The differential equations governing the system are given as follows:

$$\frac{dI_{11}(t)}{dt} = (k-1)(a+b_1 t) - \theta I_{11}(t), \qquad\qquad 0 \leq t \leq t_u \qquad (1)$$

$$\frac{dI_{12}(t)}{dt} = (k-1)\{a+b_1 t_u\} - \theta I_{12}(t), \qquad\qquad t_u \leq t \leq t_v \qquad (2)$$

$$\frac{dI_{13}(t)}{dt} = (k-1)(a-b_2 t) - \theta I_{13}(t), \qquad\qquad t_v \leq t \leq t_r \qquad (3)$$

$$\frac{dI_2(t)}{dt} = -(a-b_2 t) - \theta I_2(t), \qquad\qquad t_r \leq t \leq T \qquad (4)$$

Solving the eq. (1)-eq. (4) using the boundary conditions, $I_{11}(0) = 0$,

$I_{12}(t_u) = I_{11}(t_u)$, $I_{13}(t_v) = I_{12}(t_v)$, $I_2(T) = 0$, We get

$$I_{11}(t) = -\frac{1}{\theta^2}\left[b_1 k - b_1 + \theta^2 e^{-\theta T}\left\{\frac{1}{\theta^2}(-b_1 k + b_1) - \frac{1}{\theta}(a-ak)\right\}\right] - \frac{1}{\theta}(a-ak+b_1 t - b_1 kt) \qquad (5)$$

$$I_{12}(t) = \frac{1}{\theta}\left[Tb_1 - a + ak - Tb_1 k - Tb_1 - Tb_1 u + \frac{1}{\theta}e^{-\theta T}e^{\theta t_u}\{b_1 - b_1 k\right.$$
$$\left. + (b_1 k - b_1 + a\theta - Tb_1\theta - ak\theta + Tb_1 k\theta + Tb_1 - Tb_1 k)e^{-\theta t_u}\} + Tb_1 k + Tb_1 ku\right] \qquad (6)$$

$$I_{13}(t) = \frac{1}{\theta^2}(b_2 k - b_2 + \theta^2 e^{\theta t_v}e^{-\theta T}(\frac{1}{\theta^2}(-b_2 k + b_2) + \frac{1}{\theta}(a_2 - a_2 k + b_2(k-1)t_v)$$
$$+ \frac{1}{\theta}(e^{-\theta t_v}(k-1)(Tb_1 - a + Tb_1 e^{\theta t_v} - ae^{\theta t_v} - Tb_1 + Tb_1 e^{\theta t_v} + Tb_1 u e^{\theta t_v})) \qquad (7)$$
$$- \frac{1}{\theta^2}b_1 e^{-\theta t_v}(e^{\theta t_u} - 1)(k-1)) - \frac{1}{\theta}(a-ak-b_2 t + b_2 kt)$$

$$I_2(t) = -\frac{1}{\theta}(a-b_2 t) - \frac{1}{\theta^2}(b_2 - (b_2 + a\theta - Tb_2\theta)) \qquad (8)$$

Present value of the holding cost is

$$HC_1 = C_2\left\{\int_0^{t_u} I_{11}(t)e^{-qt}dt + \int_{t_u}^{t_v} I_{12}(t)e^{-qt}dt + \int_{t_v}^{t_r} I_{13}(t)e^{-qt}dt + \int_{t_r}^{T} I_2(t)e^{-qt}dt\right\} \qquad (9)$$

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Present value of the production cost is

$$PC_1 = \int_0^{t_u} \left(R + \frac{G}{k(a+b_1t)} \right) k(a+b_1t)e^{-qt}dt + \int_{t_u}^{t_v} \left(R + \frac{G}{k\{a+b_1t_u\}} \right) k\{a+b_1t_u\}e^{-qt}dt$$

$$+ \int_{t_v}^{t_r} \left(R + \frac{G}{k(a-b_2t)} \right) k(a-b_2t)e^{-qt}dt \qquad (10)$$

Present value of the sales revenue is

$$SR_1 = S_0 \left\{ \int_0^{t_u} (a+b_1t)e^{-qt}dt + \int_{t_u}^{t_v} (a+b_1t_u)e^{-qt}dt + \int_{t_v}^{T} (a-b_2t)e^{-qt}dt \right\} \qquad (11)$$

Present value of the total profit per unit time of the system is

$$TP_1 = \frac{1}{T}\left(SR_1 - PC_1 - HC_1 - C_1 \right) \qquad (12)$$

3.2.   Case (II)   When $t_r < t_v < T$

The differential equations governing the system are given as follows:

$$\frac{dI_{11}(t)}{dt} = (k-1)(a+b_1t) - \theta I_{11}(t), \qquad 0 \le t \le t_u \qquad (13)$$

$$\frac{dI_{12}(t)}{dt} = (k-1)\{a+b_1t_u\} - \theta I_{12}(t), \qquad t_u \le t \le t_r \qquad (14)$$

$$\frac{dI_{21}(t)}{dt} = -\{a+b_1t_u\} - \theta I_{21}(t), \qquad t_r \le t \le t_v \qquad (15)$$

$$\frac{dI_{22}(t)}{dt} = -(a-b_2t) - \theta I_{22}(t), \qquad t_r \le t \le T \qquad (16)$$

Solving the eq. (13)-eq. (16) using the boundary conditions, $I_{11}(0) = 0$,

$I_{12}(t_u) = I_{11}(t_u)$, $I_{21}(t_r) = I_{22}(t_r)$, $I_{22}(T) = 0$

$$I_{11}(t) = -\frac{1}{\theta^2}\left[b_1k - b_1 + \theta^2 e^{-\theta T}\left\{ \frac{1}{\theta^2}(-b_1k+b_1) - \frac{1}{\theta}(a-ak) \right\} \right] - \frac{1}{\theta}(a-ak+b_1t-b_1kt) \qquad (17)$$

$$I_{12}(t) = \frac{1}{\theta}\left[Tb_1 - a + ak - Tb_1k - Tb_1 - Tb_1u + \frac{1}{\theta}e^{-\theta T}e^{\theta t_u}\{b_1 - b_1k \right.$$
$$\left. +(b_1k - b_1 + a\theta - Tb_1\theta - ak\theta + Tb_1k\theta + Tb_1 - Tb_1k)e^{-\theta t_u}\} + Tb_1k + Tb_1ku \right] \qquad (18)$$

$$I_{21}(t) = -\frac{1}{\theta}(-Tb_1 + a + Tb_1 + Tb_1u - \frac{1}{\theta}(e^{-\theta T}e^{\theta t_r}(b_1e^{-\theta t_r}e^{\theta t_u} + ak\theta - b_1e^{-\theta t_r} + b_1ke^{-\theta t_r} + a\theta e^{-\theta t_r}$$
$$-b_1ke^{\theta t_u - \theta t_r} - Tb_1k\theta + Tb_1k\theta + Tb_1k\theta u - Tb_1\theta e^{-\theta t_r} - ak\theta e^{-\theta t_r} + Tb_1k\theta e^{-\theta t_r} + Tb_1\theta e^{-\theta t_r} - Tb_1k\theta e^{-\theta t_r})) \qquad (19)$$

$$I_{22}(t) = -\frac{1}{\theta}(a - b_2 t) - \frac{1}{\theta^2}(b_2 - (b_2 + a\theta - Tb_2\theta)) \tag{20}$$

Present value of the holding cost is

$$HC_2 = C_2 \left\{ \int_0^{t_u} I_{11}(t)e^{-qt}dt + \int_{t_u}^{t_r} I_{12}(t)e^{-qt}dt + \int_{t_r}^{t_v} I_{21}(t)e^{-qt}dt + \int_{t_v}^{T} I_{22}(t)e^{-qt}dt \right\} \tag{21}$$

Present value of the production cost is

$$PC_2 = \int_0^{t_u} \left(R + \frac{G}{k(a + b_1 t)} \right) k(a + b_1 t)e^{-qt}dt + \int_{t_u}^{t_r} \left(R + \frac{G}{k\{a + b_1 t_u\}} \right) k\{a + b_1 t_u\}e^{-qt}dt \tag{22}$$

Present value of the sales revenue is

$$SR_2 = S_0 \left\{ \int_0^{t_u} (a + b_1 t)e^{-qt}dt + \int_{t_u}^{t_v} (a + b_1 t_u)e^{-qt}dt + \int_{t_v}^{T} (a - b_2 t)e^{-qt}dt \right\} \tag{23}$$

Present value of the total profit per unit time of the system is

$$TP_2 = \frac{1}{T}(SR_2 - PC_2 - HC_2 - C_1) \tag{24}$$

3.3. Case (III)   When $t_r < t_u < T$

The differential equations governing the system are given as follows:

$$\frac{dI_1(t)}{dt} = (k-1)(a + b_1 t) - \theta I_1(t), \qquad\qquad 0 \le t \le t_r \tag{25}$$

$$\frac{dI_{21}(t)}{dt} = -(a + b_1 t) - \theta I_{21}(t), \qquad\qquad t_r \le t \le t_u \tag{26}$$

$$\frac{dI_{22}(t)}{dt} = -\{a + b_1 t_u\} - \theta I_{22}(t), \qquad\qquad t_u \le t \le t_v \tag{27}$$

$$\frac{dI_{23}(t)}{dt} = -(a - b_2 t) - \theta I_{23}(t), \qquad\qquad t_v \le t \le T \tag{28}$$

Solving the eq. (25)-eq. (28) using the boundary conditions, $I_{11}(0) = 0$, $I_{21}(t_r) = I_1(t_r)$, $I_{21}(t_u) = I_{22}(t_u)$, $I_{23}(T) = 0$

$$I_{11}(t) = -\frac{1}{\theta^2}\left[b_1 k - b_1 + \theta^2 e^{-\theta T}\left\{ \frac{1}{\theta^2}(-b_1 k + b_1) - \frac{1}{\theta}(a - ak) \right\} \right] - \frac{1}{\theta}(a - ak + b_1 t - b_1 kt) \tag{29}$$

$$I_{21}(t) = b_1 - \theta^2 e^{-\theta T} e^{\theta t_r} (\frac{1}{\theta}(a - ak + b_1 t_r - b_1 kt_r) - \frac{1}{\theta}(a + b_1 t_r) + \frac{b}{\theta^2}$$

$$+ \frac{1}{\theta^2}(b_1 k - b_1 + \theta^2 e^{\theta t_r}(\frac{1}{\theta^2}(-b_1 k + b_1) - \frac{1}{\theta^2}(a - ak) - \frac{1}{\theta}(a + b_1 t) \tag{30}$$

$$I_{22}(t) = -\frac{1}{\theta}(a + b_1 t_u - \frac{1}{\theta}(e^{-\theta T}e^{\theta t_u}(b_1 + (-b_1 + b_1 k + a\theta - ak\theta + Tb_1 k - Tb_1 k\theta)e^{-\theta t_u}$$

$$+(b_1 k + ak\theta - Tb_1 k\theta + b_1 k\theta t_r + Tb_1 k\theta)e^{\theta t_r}e^{-\theta t_u})))$$

(31)

$$I_{23}(t) = -\frac{1}{\theta}(a - b_2 t) - \frac{1}{\theta^2}(b_2 - (b_2 + a\theta - Tb_2\theta))$$

(32)

Present value of the holding cost is

$$HC_3 = C_2\left\{\int_0^{t_r} I_1(t)e^{-qt}dt + \int_{t_r}^{t_u} I_{21}(t)e^{-qt}dt + \int_{t_u}^{t_v} I_{22}(t)e^{-qt}dt + \int_{t_v}^{T} I_{23}(t)e^{-qt}dt\right\}$$

(33)

Present value of the production cost is

$$PC_3 = \int_0^{t_r}\left(R + \frac{G}{k(a + b_1 t)}\right)k(a + b_1 t)e^{-qt}dt$$

(34)

Present value of the sales revenue is

$$SR_3 = S_0\left\{\int_0^{t_u}(a + b_1 t)e^{-qt}dt + \int_{t_u}^{t_v}(a + b_1 t_u)e^{-qt}dt + \int_{t_v}^{T}(a - b_2 t)e^{-qt}dt\right\}$$

(35)

Present value of the total profit per unit time of the system is

$$TP_3 = \frac{1}{T}(SR_3 - PC_3 - HC_3 - C_1)$$

(36)

Our objective is to find maximum total profit per unit time in each case, i.e.,

$$\max TP_i(r) = \frac{1}{T}(SR_i - PC_i - HC_i - C_1) \text{ such that } 0 < r < 1, i = 1, 2, 3$$

4. Solution Procedure

The solution procedure is as follows:

Step 1. Equate the first derivatives of TP_m in eq. (12), (24) and (36), denoted by $TP_m(r)$ with respect to r to zero and solve all the three equations for r.

Step 2. Check for concavity. The sufficient condition for maximum $TP_m(r)$ is $\frac{d^2 TP_m(r)}{dr^2} < 0$.

Step 3. If $0 \le r \le 1$, calculate TP_m from eq. (12), (24) and (36).

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5.   Numerical Example and Sensitivity Analysis

We determine the optimal value of decision variables and net profit using solution procedure defined in last section. We use the following values of the parameters in appropriate units: $S_o=100$, $T=12$, $k=1.5$, $C_1=100$, $C_2=1$, $a=10$, $b_1=b_2=0.1$, $\theta=0.01$, $R=10$, $G=250$, $q=0.6$, $t_u=3$, $t_v=6$. The optimal net profit TP_m^* and optimal time parameter for production run time t_r^* for the three cases are provided in the following table:

Table 1:   Summary of results

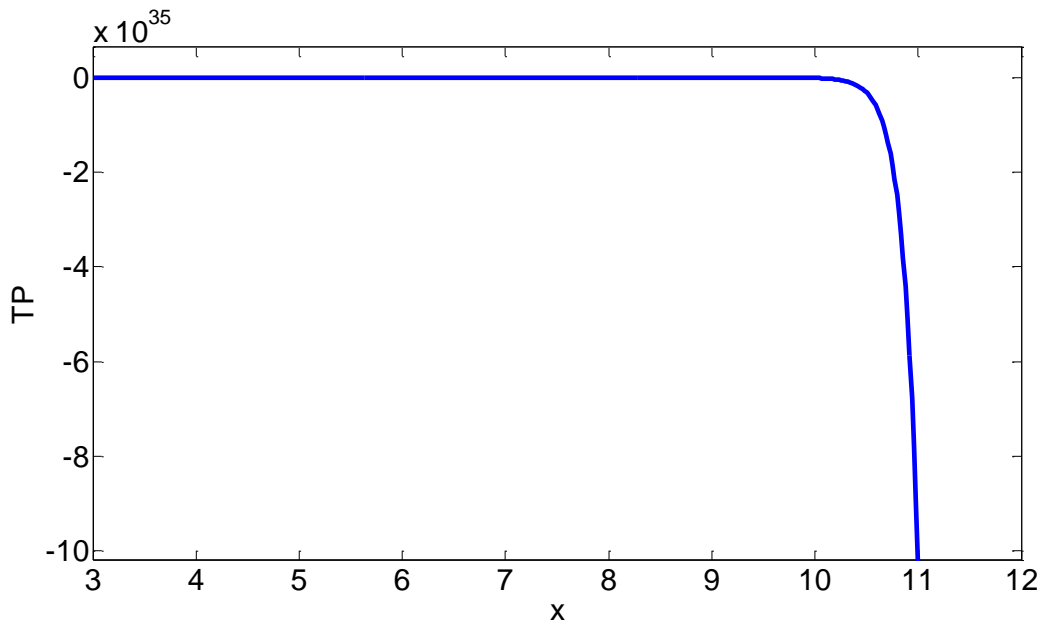| *Case*	r^*	TP_m^*
I	3	4.959e+20
II	3	8.688e+20
III	6	3.51e+20

Fig.1.   Net profit in Case (I)

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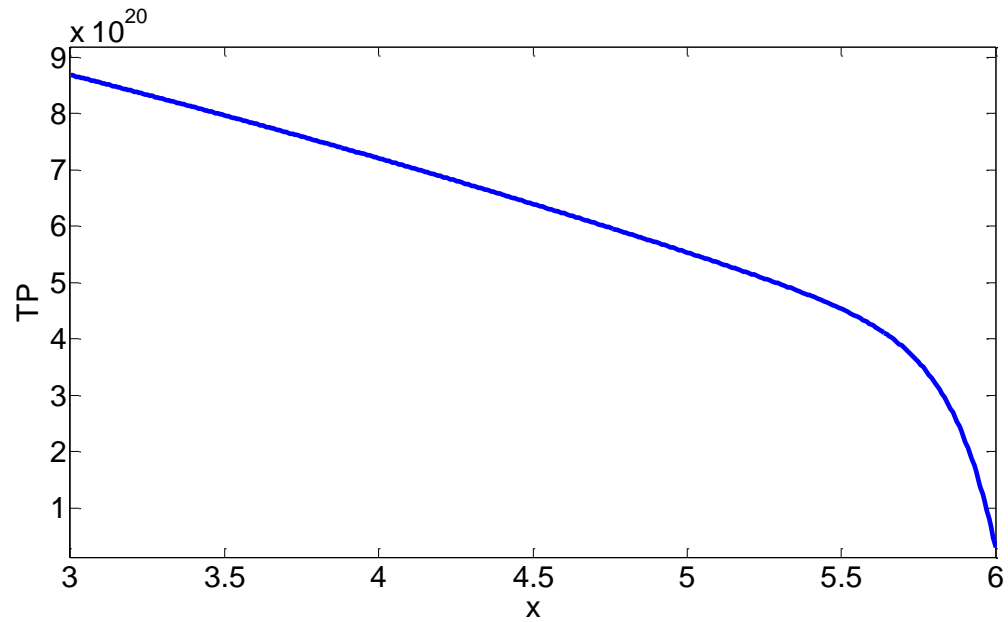**Pak.j.stat.oper.res. Vol.IX No.4 2013 pp427-440**
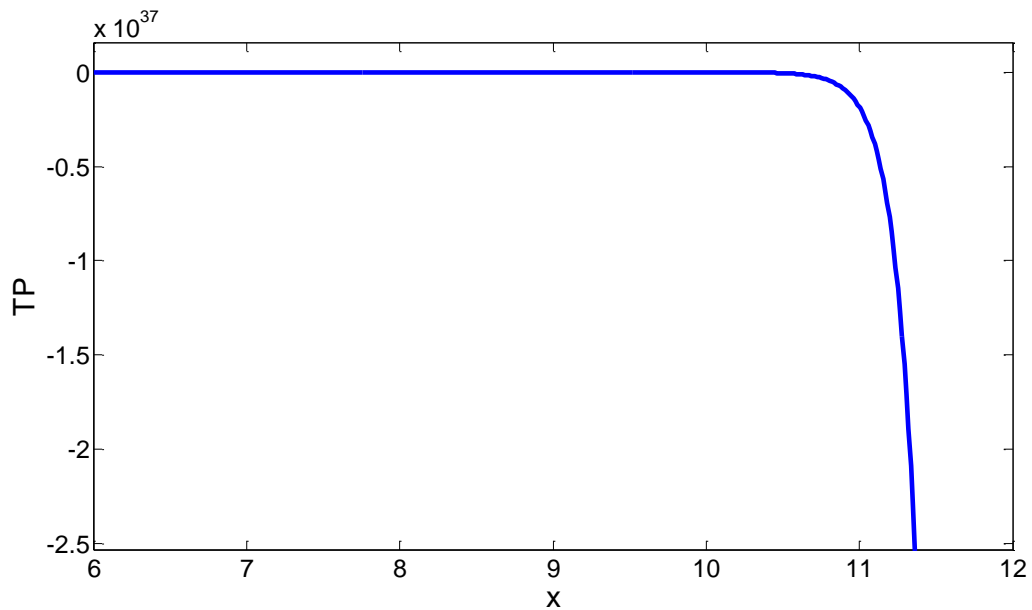
Fig.2.   Net profit in Case (II)

Fig. 3. Net profit in Case (III)

Based on the above experimental outcomes, sensitivity analysis is provided for Case II for which the maximum profit has achieved. It is performed by altering the parameters by 50% and 25%, taking one parameter at a time while all other parameters being constant (See, Table 2).

Table 2: Sensitivity analyses

Parameter	Value	TP_m
S_o	50	4.424e+20
	75	6.556e+20
	125	1.082e+21
	150	1.295e+21
a	5	3.943e+20
	7.5	6.316e+20
	12.5	1.106e+21
	15	1.343e+21
b_1	0.05	6.907e+20
	0.075	7.798e+20
	0.125	9.579e+20
	0.15	1.047e+21
k	0.75	1.022e+21
	1.125	9.456e+20
	1.875	7.92e+20
	2.25	7.152e+20
θ	0.005	8.321e+20
	0.0075	8.519e+20
	0.0125	8.832e+20
	0.015	8.954e+20
t_u	1.5	6.869e+20
	2.25	7.768e+20
	3.75	9.632e+20
	4.5	1.06e+21
t_v	3	-2.978e+10
	4.5	1.169e+16
	7.5	5.703e+25
	9	3.501e+30

The observations made from sensitivity analysis can be described as the following:

1. Total profit increases with the rising selling price S_o which is a fairly usual occurrence.

2. Total profit intensifies with the rising initial parameter '*a*' and time sensitive parameter '*b_1*'of demand. Elevated demand of manufactured units stimulates more production which becomes a important cause to make additional profit.

3. With the increase in the coefficient related to production rate '*k*', total profit decreases. Quantity of production becomes more in proportion to the demand for higher values of '*k*'.

4. Total profit becomes lesser with increasing value of the deterioration rate '*θ*'. The larger quantity of deteriorated units leads to loss of sales that could have been added to revenue.

5. As the time points 't_u' and 't_v' increase, total profit increases. Since, 't_u' is the point when demand stabilizes and 't_v' is the point till the demand remains stable, hence their increasing values leads to the greater demand which causes higher turnover.

6. Conclusion

This article considers the particular items following trapezoidal ramp type demand incorporating inflation and time value of money. Three cases according to the demand have been discussed and for each case, a profit function is formulated for maximization. The condition to find the optimal solution is given and using numerical experiment, we show that the profit is maximum when the demand stabilizes before or with the production stops. Sensitivity analysis is performed corresponding to various parameters and the results confirm the authenticity of the model under study. The proposed study can be extended in various dimensions including stochastic or fuzzy modelling.

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